

Health Equity Assessment Toolkit

Built-in Database Edition

TECHNICAL NOTES

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1. Introduction

The Health Equity Assessment Toolkit (HEAT) enables the assessment of within-country inequalities, i.e. inequalities that exist between population subgroups within a country, based on disaggregated data and summary measures of inequality. Disaggregated data show the level of health by population subgroup of a given dimension of inequality. Summary measures build on disaggregated data and present the degree of inequality across multiple population subgroups in a single numerical figure. These technical notes provide information about the disaggregated data (section 2) and the summary measures (section 3) presented in HEAT.

2. Disaggregated data

HEAT enables the assessment of inequalities using disaggregated data, i.e. data broken down by population subgroups, from the [WHO Health Equity Monitor database](#) (2016 update). The database currently contains over 30 reproductive, maternal, newborn and child health (RMNCH) indicators, disaggregated by five dimensions of inequality. Data are based on re-analysis of more than 280 Demographic and Health Surveys (DHS) and Multiple Indicator Cluster Surveys (MICS) conducted in 102 countries between 1993 and 2014. For almost three quarters of the countries, data are available for at least two time points (i.e. multiple rounds of data exist). A full list of study countries, with corresponding ISO3 country codes and information about survey source(s) and year(s) is given in Supplementary table 1.

Micro-level DHS and MICS data were analysed by the International Center for Equity in Health based in the Federal University of Pelotas, Brazil. Survey design specifications were taken into consideration during the analysis. The same methods of calculation for data analysis were applied across all surveys to generate comparable estimates across countries and over time. Estimates of disaggregated data are presented alongside 95% confidence intervals, and the population share of the subgroup. The population share for each indicator is the percentage of the affected population – the indicator denominator – represented by the subgroup in a given country.

2.1 Health indicators

Table 1 lists the RMNCH indicators currently available in the WHO Health Equity Monitor database. Detailed information about the criteria used to calculate the numerator and denominator values for each indicator are available in the indicator compendium or in the WHO Indicator and Measurement Registry, under the topic Health Equity Monitor (www.who.int/gho/indicator_registry/en/).

Table 1 Health indicators

Indicator name	Indicator abbreviation
Favourable health intervention indicators	
Antenatal care coverage – at least four visits (in the two or three years preceding the survey) (%)	anc4
Antenatal care coverage – at least four visits (in the five years preceding the survey) (%)	anc45
Antenatal care coverage – at least one visit (in the two or three years preceding the survey) (%)	anc1
Antenatal care coverage – at least one visit (in the five years preceding the survey) (%)	anc15
BCG immunization coverage among one-year-olds (%)	bcbv
Births attended by skilled health personnel (in the two or three years preceding the survey) (%)	sba
Births attended by skilled health personnel (in the five years preceding the survey) (%)	sba5
Births by caesarean section (in the two or three years preceding the survey) (%)*	csection

Births by caesarean section (in the five years preceding the survey) (%)*	csection5
Children aged < 5 years sleeping under insecticide-treated nets (%)	itnch
Children aged < 5 years with diarrhoea receiving oral rehydration salts (%)	ors
Children aged < 5 years with diarrhoea receiving oral rehydration therapy and continued feeding (%)	ort
Children aged < 5 years with pneumonia symptoms taken to a health facility (%)	carep
Children aged 6–59 months who received vitamin A supplementation (%)	vita
Composite coverage index (%)	cci
Contraceptive prevalence – modern and traditional methods (%)	cpmt
Contraceptive prevalence – modern methods (%)	cpmo
Demand for family planning satisfied (%)	fps
DTP3 immunization coverage among one-year-olds (%)	dptv
Early initiation of breastfeeding (in the two or three years preceding the survey) (%)	ebreast
Early initiation of breastfeeding (in the five years preceding the survey) (%)	ebreast5
Full immunization coverage among one-year-olds (%)	fullv
Measles immunization coverage among one-year-olds (%)	mslv
Polio immunization coverage among one-year-olds (%)	poliov
Pregnant women sleeping under insecticide-treated nets (%)	itnwm
Adverse health outcome indicators	
Adolescent fertility rate (per 1000 women aged 15–19 years)**	asfr1
Infant mortality rate (deaths per 1000 live births)	imr
Neonatal mortality rate (deaths per 1000 live births)	nmr
Obesity prevalence in non-pregnant women aged 15–49 years, BMI \geq 30 (%)	obesewm
Stunting prevalence in children aged < 3 years (%)	stunt3
Stunting prevalence in children aged < 5 years (%)	stunt5
Total fertility rate (per woman)**	tfr
Under-five mortality rate (deaths per 1000 live births)	u5mr
Underweight prevalence in children aged < 3 years (%)	uweight3
Underweight prevalence in children aged < 5 years (%)	uweight5
Wasting prevalence in children aged < 3 years (%)	wast3
Wasting prevalence in children aged < 5 years (%)	wast5

*Note that the indicators “Births by caesarean section (in the two or three years preceding the survey)” and “Births by caesarean section (in the five years preceding the survey)” are treated as favourable health intervention indicators, even though the maximum level may not be the most desirable situation (as is the case for other favourable health intervention indicators, such as full immunization coverage).

**Note that the indicators “Adolescent fertility rate” and “Total fertility rate” are treated as adverse health outcome indicators, even though the minimum level may not be the most desirable situation (as is the case for other adverse outcome indicators, such as infant mortality rate).

As indicated in table 1, health indicators can be divided into favourable and adverse health indicators. Favourable health indicators measure desirable health events that are promoted through public health action. They include health intervention indicators, such as antenatal care coverage, and desirable health outcome indicators, such as life expectancy. For these indicators, the ultimate goal is to achieve a maximum level, either in health intervention coverage or health outcome (for example, complete coverage of antenatal care or the highest possible life expectancy). Adverse health indicators, on the other hand, measure undesirable events, that are to be reduced or eliminated through public health action. They include undesirable health outcome indicators, such as stunting prevalence in children aged less than five years or under-five mortality rate. Here, the ultimate goal is

to achieve a minimum level in health outcome (for example, a stunting prevalence or mortality rate of zero).

In the WHO Health Equity Monitor database, all health intervention indicators are favourable health indicators and all health outcome indicators are adverse health indicators. This differentiation is important as the type of indicator has implications for the calculation of summary measures (see section 2).

2.2 Dimensions of inequality

Health indicators from the WHO Health Equity Monitor database were disaggregated by five dimensions of inequality: economic status, education, place of residence, subnational region and child's sex (where applicable).

Economic status was determined using a wealth index. Country-specific indices were based on owning selected assets and having access to certain services, and constructed using principal component analysis. Within each country the index was divided into quintiles of households, thereby creating five equal subgroups that each account for 20% of the population. Note that certain indicators have denominator criteria that do not include all households and/or are more likely to include households from a specific quintile; thus the quintile share of the population for a given indicator may not equal 20%. For example, there are often more live births reported by the poorest quintile than the richest quintile, resulting in the poorest quintile representing a larger share of the population for indicators such as the coverage of births attended by skilled health personnel.

Education refers to the highest level of schooling attained by the woman (or the mother, in the case of newborn and child health interventions, child malnutrition and child mortality): no education, primary school, or secondary school or higher. These levels reflect the highest level of schooling ever attended by the woman.

For **place of residence** and **subnational region**, country-specific criteria for place of residence and subnational region were applied.

Table 2 lists the five dimensions of inequality available in the WHO Health Equity Monitor database along with their basic characteristics.

Table 2 Dimensions of inequality

Dimension of inequality	Number of subgroups	Ordered subgroups
Economic status	More than two subgroups	Yes
Education	More than two subgroups	Yes
Place of residence	Two subgroups	-
Sex	Two subgroups	-
Subnational region	More than two subgroups	No

At the most basic level, dimensions of inequality can be divided into dimensions that compare the situation in two population subgroups (e.g. girls and boys) versus dimensions that look at the situation in more than two population subgroups (e.g. economic status quintiles).

In the case of dimensions with more than two population subgroups it is possible to differentiate between dimensions with ordered subgroups and non-ordered subgroups. Ordered subgroups have an inherent positioning and can be ranked. For example, education has an inherent ordering of subgroups in the sense that those with less education unequivocally have *less* of something compared to those with more education. Non-ordered subgroups, by contrast, are not based on criteria that can be logically ranked. Subnational regions are an example of non-ordered groupings.

These characteristics (number of subgroups and ordered vs. non-ordered subgroups) are important as they impact on the calculation of summary measures (see section 2).

3. Summary measures

HEAT enables the assessment of inequalities using multiple summary measures of inequality. Summary measures are calculated based on disaggregated data from the [WHO Health Equity Monitor database](#) (2016 update), combining estimates of a given health indicator for two or more subgroups into a single numerical figure. Table 3 lists the 15 summary measures currently available in HEAT along with their basic characteristics.

Table 3 Summary measures of inequality

Summary measure name	Summary measure abbreviation	Simple vs. complex measure	Ordered vs. non-ordered complex measure	Weighted vs. unweighted measure
Absolute measures				
Absolute concentration index	aci	Complex	Ordered	Weighted
Between-group variance	bgv	Complex	Non-ordered	Weighted
Difference	d	Simple	-	Unweighted
Mean difference from best performing subgroup	mdb	Complex	Non-ordered	Weighted
Mean difference from mean	mdm	Complex	Non-ordered	Weighted
Population attributable risk	par	Complex	Non-ordered	Weighted
Slope index of inequality	sii	Complex	Ordered	Weighted
Relative measures				
Index of disparity	idis	Complex	Non-ordered	Unweighted
Kunst-Mackenbach index	kmi	Complex	Ordered	Weighted
Mean log deviation	mld	Complex	Non-ordered	Weighted
Population attributable fraction	paf	Complex	Non-ordered	Weighted
Ratio	r	Simple	-	Unweighted
Relative concentration index	rci	Complex	Ordered	Weighted
Relative index of inequality	rii	Complex	Ordered	Weighted
Theil index	ti	Complex	Non-ordered	Weighted

As indicated in table 3, summary measures of inequality can be divided into absolute measures and relative measures. For a given health indicator, absolute inequality measures indicate the magnitude of difference in health between subgroups. They retain the same unit as the health indicator.¹ Relative inequality measures, on the other hand, show proportional differences in health among subgroups and have no unit.

Furthermore, it is possible to differentiate between simple and complex measures of inequality. Simple measures make pairwise comparisons between two subgroups, such as the most and least wealthy. They can be calculated for all health indicators and dimensions of inequality. The characteristics of the indicator and dimension determine which two subgroups are compared to assess inequality. Contrary to simple measures, complex measures make use of data from all subgroups to assess inequality. They can be calculated for all health indicators, but they can only be calculated for dimensions with more than two subgroups.²

¹ One exception to this is the between-group variance (BGV), which takes the squared unit of the health indicator.

² Exceptions to this are the population attributable risk (PAR) and the population attributable fraction (PAF), which can be calculated for all dimensions of inequality.

Complex measures can further be divided into ordered complex measures and non-ordered complex measures of inequality. Ordered measures can only be calculated for dimensions with more than two subgroups that have a natural ordering. Here, the calculation is also influenced by the type of indicator (favourable vs. adverse). Non-ordered measures are only calculated for dimensions with more than two subgroups that have no natural ordering.³

Finally, summary measures may be weighted or unweighted. Weighted measures take into account the population size of each subgroup, while unweighted measures treat each subgroup as equally sized. Importantly, simple measures are always unweighted and complex measures may be weighted or unweighted.

The following sections give further information about the definition, calculation and interpretation of each summary measure of inequality. An overview of the formulae and characteristics is provided in Supplementary table 2. Further information about summary measures of inequality can be found in the *Handbook on health inequality monitoring: with a special focus on low- and middle-income countries*.⁴

3.1 Absolute measures

3.1.1 Absolute concentration index

Definition

The absolute concentration index (ACI) is a complex measure of inequality that shows the health gradient across multiple subgroups with natural ordering, on an absolute scale. It indicates the extent to which a health indicator is concentrated among the disadvantaged or the advantaged.

Calculation

To calculate ACI, a weighted sample of the whole population is ranked from the most-disadvantaged subgroup (at rank zero or 0) to the most-advantaged subgroup (at rank 1). Based on this ranking, ACI can be calculated as:

$$(1) \quad ACI = \sum_j p_j (2X_j - 1) y_j,$$

where y_j indicates the health indicator estimate for subgroup j , p_j the population share of subgroup j and X_j the relative rank of subgroup j , which is defined as: $X_j = \sum_j p_j - 0.5p_j$.

ACI is calculated for dimensions with more than two subgroups that have a natural ordering. It is missing if at least one subgroup estimate is missing.

Interpretation

If there is no inequality, ACI takes the value zero. Positive values indicate a concentration of the health indicator among the advantaged, while negative values indicate a concentration of the health indicator among the disadvantaged. The larger the absolute value of ACI, the higher the level of inequality.

3.1.2 Between-group variance

Definition

³Non-ordered complex measures could also be calculated for dimensions with ordered subgroups, however, in practice, they are not used for such dimensions and are therefore only reported for dimensions with non-ordered subgroups.

⁴ World Health Organization (2013). *Handbook on health inequality monitoring: with a special focus on low- and middle-income countries*. Geneva: World Health Organization. Available from: www.who.int/gho/health_equity/handbook/en/

The between-group variance (BGV) is a complex measure of inequality that takes into account the population share of each subgroup.

Calculation

BGV is calculated as the weighted sum of squared differences between the subgroup estimates y_j and the national average μ . Squared differences are weighted by each subgroup's population share p_j :

$$(2) \quad BGV = \sum_j p_j (y_j - \mu)^2.$$

BGV is calculated for dimensions with more than two subgroups that have no natural ordering. It is missing if at least one subgroup estimate is missing.

Interpretation

BGV takes only positive values with larger values indicating higher levels of inequality. BGV is zero if there is no inequality. BGV is more sensitive to outlier estimates as it gives more weight to the estimates that are further from the national average.

3.1.3 Difference

Definition

The difference (D) is a simple measure of inequality that shows the absolute inequality between two subgroups, without taking into consideration their population share.

Calculation

D is calculated as the difference between two subgroups. For dimensions with more than two subgroups that have a natural ordering (e.g. education), the most-advantaged and most-disadvantaged subgroups are compared, while for dimensions with more than two subgroups that have no natural ordering (e.g. subnational region), the subgroups with the highest and lowest estimates are used:

$$(3) \quad D = y_{max} - y_{min}.$$

Note that the selection of y_{max} and y_{min} depends on the type of health indicator and on the characteristics of the dimension of inequality, for which D is calculated.⁵ For place of residence, D is calculated as the difference between urban and rural areas in the case of favourable health intervention indicators and as the difference between rural and urban areas in the case of adverse health outcome indicators. For sex, D is calculated as the difference between females and males in the case of favourable health intervention indicators and as the difference between males and females in the case of adverse health outcome indicators. For dimensions with more than two subgroups that have a natural ordering y_{max} refers to the most-advantaged subgroup and y_{min} to the most-disadvantaged subgroup in the case of favourable health intervention indicators, whereas y_{max} refers to the most-disadvantaged subgroup and y_{min} to the most-advantaged subgroup in the case of adverse health outcome indicators. For dimensions with more than two subgroups that have no natural ordering, the lowest estimate is subtracted from the highest estimate, regardless of the health indicator type.

D is calculated for all dimensions of inequality. In the case of dimensions with two subgroups or in the case of dimensions with more than two subgroups that have no natural ordering, D is missing if at least one subgroup estimate is missing. In the case of dimensions with more than two subgroups that have a natural ordering, D is missing if the estimates for the most-advantaged and/or most-disadvantaged subgroup are missing.

⁵ Selections were made based on convenience of data interpretation (that is, providing positive values for difference calculations). In the case of sex, the selection does not represent an assumed advantage of one sex over the other.

Interpretation

If there is no inequality, D takes the value zero. Greater absolute values indicate higher levels of inequality. For favourable health intervention indicators, positive values indicate higher coverage in the advantaged subgroups and negative values indicate higher coverage in the disadvantaged subgroups. For adverse health outcome indicators, positive values indicate a higher concentration of the indicator among the disadvantaged and negative values indicate a higher concentration among the advantaged.

3.1.4 Mean difference from best performing subgroup

Definition

The mean difference from best performing subgroup (MDB) is a complex measure of inequality that shows the difference between each subgroup and the best performing subgroup, on average.

Calculation

MDB is calculated as the weighted sum of absolute differences between the subgroup estimates y_j and the estimate for the reference group y_{ref} . Absolute differences are weighted by each subgroup's population share p_j :

$$(4) \quad MDB = \sum_j p_j |y_j - y_{ref}|.$$

Note that the selection of the reference subgroup y_{ref} depends on the type of health indicator and on the characteristics of the dimension of inequality, for which MDB is calculated. For place of residence, urban is selected as the reference group, regardless of the health indicator type. For dimensions with more than two subgroups that have a natural ordering (e.g. education), y_{ref} refers to the most-advantaged subgroup, regardless of the health indicator type. For sex and for dimensions with more than two subgroups that have no natural ordering (e.g. subnational region), y_{ref} refers to the subgroup with the highest estimate in the case of favourable health intervention indicators and to the subgroup with the lowest estimate in the case of adverse health outcome indicators.

MDB is calculated for dimensions with more than two subgroups that have no natural ordering. It is missing if at least one subgroup estimate is missing. Note that the analytic 95% confidence intervals for MDB are not calculated.

Interpretation

MDB takes only positive values with larger values indicating higher levels of inequality. MDB is zero if there is no inequality.

3.1.5 Mean difference from mean

Definition

The mean difference from mean (MDM) is a complex measure of inequality that shows the difference between each subgroup and the national level, on average.

Calculation

MDB is calculated as the weighted sum of absolute differences between the subgroup estimates y_j and the national average μ . Absolute differences are weighted by each subgroup's population share p_j :

$$(5) \quad MDM = \sum_j p_j |y_j - \mu|.$$

MDM is calculated for dimensions with more than two subgroups that have no natural ordering. It is missing if at least one subgroup estimate is missing. Note that the analytic 95% confidence intervals for MDM are not calculated.

Interpretation

MDM takes only positive values with larger values indicating higher levels of inequality. MDM is zero if there is no inequality.

3.1.6 Population attributable risk

Definition

The population attributable risk (PAR) is a complex measure of inequality that shows the potential for improvement in the national level of a health indicator that could be achieved if all subgroups had the same level of health as a reference subgroup.

Calculation

PAR is calculated as the difference between the estimate for the reference subgroup y_{ref} and the national average μ :

$$(6) \quad PAR = y_{ref} - \mu.$$

Note that the selection of the reference subgroup y_{ref} depends on the type of health indicator and on the characteristics of the dimension of inequality, for which PAR is calculated. For place of residence, urban is selected as the reference group, regardless of the health indicator type. For dimensions with more than two subgroups that have a natural ordering (e.g. education), y_{ref} refers to the most-advantaged subgroup, regardless of the health indicator type. For sex and for dimensions with more than two subgroups that have no natural ordering (e.g. subnational region), y_{ref} refers to the subgroup with the highest estimate in the case of favourable health intervention indicators and to the subgroup with the lowest estimate in the case of adverse health outcome indicators.

PAR is calculated for all dimensions. In the case of place of residence and in the case of dimensions with more than two subgroups that have a natural ordering, PAR is missing if the estimate for the reference subgroup is missing. In the case of sex and in the case of dimensions with more than two subgroups that have no natural ordering, PAR is missing if at least one subgroup estimate is missing.

Interpretation

PAR takes positive values for favourable health intervention indicators and negative values for adverse health outcome indicators. The larger the absolute value of PAR, the higher the level of inequality. PAR is zero if no further improvement can be achieved, i.e. if all subgroups have reached the same level of health as the reference group.

3.1.7 Slope index of inequality

Definition

The slope index of inequality (SII) is a complex measure of inequality that represents the absolute difference in predicted values of a health indicator between the most-advantaged and most-disadvantaged (or vice versa for adverse health outcome indicators), while taking into consideration all the other subgroups – using an appropriate regression model.

Calculation

To calculate SII, a weighted sample of the whole population is ranked from the most-disadvantaged subgroup (at rank zero or 0) to the most-advantaged subgroup (at rank 1). This ranking is weighted,

accounting for the proportional distribution of the population within each subgroup. The population of each subgroup is then considered in terms of its range in the cumulative population distribution, and the midpoint of this range. Then, the health indicator of interest is regressed against this midpoint value using a generalized linear model with logit link, and the predicted values of the health indicator are calculated for the two extremes (rank 1 and rank 0).

For favourable health intervention indicators, the difference between the predicted values at rank 1 (v_1) and rank 0 (v_0) (covering the entire distribution) generates the SII value:

$$(7a) \quad SII = v_1 - v_0.$$

For adverse health outcome indicators, the calculation is reversed and the SII value is calculated as the difference between the predicted values at rank 0 (v_0) and rank 1 (v_1) (covering the entire distribution):

$$(7b) \quad SII = v_0 - v_1.$$

SII is calculated for dimensions with more than two subgroups that have a natural ordering. It is missing if at least one subgroup estimate is missing. Note that the bootstrap 95% confidence intervals for SII are not calculated.

Interpretation

If there is no inequality, SII takes the value zero. Greater absolute values indicate higher levels of inequality. For favourable health intervention indicators, positive values indicate higher coverage in the advantaged subgroups and negative values indicate higher coverage in the disadvantaged subgroups. For adverse health outcome indicators, positive values indicate a higher concentration of the indicator among the disadvantaged and negative values indicate a higher concentration among the advantaged.

3.2 Relative measures

3.2.1 Index of disparity

Definition

The index of disparity (IDIS) is a complex measure of inequality that shows the proportional difference between each subgroup and the national level, on average.

Calculation

IDIS is calculated as the absolute sum of differences between the subgroup estimates y_j and the national average μ , divided by the national average μ :

$$(8) \quad IDIS = \frac{1}{n} * \frac{\sum_j |y_j - \mu|}{\mu} * 100,$$

where y_j indicates the estimate for subgroup j , n the number of subgroups and μ the national average.

IDIS is calculated for dimensions with more than two subgroups that have no natural ordering. It is missing if at least one subgroup estimate is missing. Note that the analytic 95% confidence intervals for IDIS are not calculated.

Interpretation

IDIS takes only positive values with larger values indicating higher levels of inequality. IDIS is zero if there is no inequality.

3.2.2 Kunst-Mackenbach index

Definition

The Kunst-Mackenbach index (KMI) is a complex measure of inequality that represents the ratio of predicted values of a health indicator of the most-advantaged to the most-disadvantaged (or vice versa for adverse health outcome indicators), while taking into consideration all the other subgroups – using an appropriate regression model.

Calculation

To calculate KMI, a weighted sample of the whole population is ranked from the most-disadvantaged subgroup (at rank zero or 0) to the most-advantaged subgroup (at rank 1). This ranking is weighted, accounting for the proportional distribution of the population within each subgroup. The population of each subgroup is then considered in terms of its range in the cumulative population distribution, and the midpoint of this range. Then, the health indicator of interest is regressed against this midpoint value using a generalized linear model with logit link, and the predicted values of the health indicator are calculated for the two extremes (rank 1 and rank 0).

For favourable health intervention indicators, the ratio of the predicted values at rank 1 (v_1) to rank 0 (v_0) (covering the entire distribution) generates the KMI value:

$$(9a) \quad KMI = v_1/v_0.$$

For adverse health outcome indicators, the calculation is reversed and the KMI value is calculated as the ratio of the predicted values at rank 0 (v_0) to rank 1 (v_1) (covering the entire distribution):

$$(9b) \quad KMI = v_0/v_1.$$

KMI is calculated for dimensions with more than two subgroups that have a natural ordering. It is missing if at least one subgroup estimate is missing. Note that the bootstrap 95% confidence intervals for KMI are not calculated.

Interpretation

If there is no inequality, KMI takes the value one. It takes only positive values (larger or smaller than one). The further the value of KMI from one, the higher the level of inequality.

3.2.3 Mean log deviation

Definition

The mean log deviation (MLD) is a complex measure of inequality that takes into account the population share of each subgroup.

Calculation

MLD is calculated as the sum of differences between the natural logarithm of the share of health of each subgroup ($\ln(\frac{y_j}{\mu})$) and the population share of each subgroup (p_j). MLD may be more easily interpreted when multiplied by 1000:

$$(10) \quad MLD = \sum_j p_j (-\ln(\frac{y_j}{\mu})) * 1000,$$

where y_j indicates the estimate for subgroup j , p_j the population share of subgroup j and μ the national average.

MLD is calculated for dimensions with more than two subgroups that have no natural ordering. It is missing if at least one subgroup estimate is missing. Note that the bootstrap 95% confidence intervals for MLD are not calculated.

Interpretation

If there is no inequality, MLD takes the value zero. Greater absolute values indicate higher levels of inequality. MLD is more sensitive to health differences further from the national average (by the use of the logarithm).

3.2.4 Population attributable fraction

Definition

The population attributable fraction (PAF) is a complex measure of inequality that shows the potential for improvement in the national level of a health indicator, in relative terms, that could be achieved if all subgroups had the same level of health as a reference subgroup.

Calculation

PAF is calculated by dividing the population attributable risk (PAR) by the national average μ and multiplying the fraction by 100:

$$(11) \quad PAF = \frac{PAR}{\mu} * 100.$$

PAF is calculated for all dimensions. In the case of place of residence and in the case of dimensions with more than two subgroups that have a natural ordering (e.g. education), PAF is missing if the estimate for the reference subgroup is missing. In the case of sex and in the case of dimensions with more than two subgroups that have no natural ordering (e.g. subnational region), PAF is missing if at least one subgroup estimate is missing.

Interpretation

PAF takes positive values for favourable health intervention indicators and negative values for adverse health outcome indicators. The larger the absolute value of PAF, the larger the degree of inequality. PAF is zero if no further improvement can be achieved, i.e. if all subgroups have reached the same level of health as the reference group.

3.2.5 Ratio

Definition

The ratio (R) is a simple measure of inequality that shows the relative inequality between two subgroups, without taking into consideration their population share.

Calculation

R is calculated as the ratio of two subgroups. For dimensions with more than two subgroups that have a natural ordering (e.g. education), the most-advantaged and most-disadvantaged subgroups are compared, while for dimensions with more than two subgroups that have no natural ordering (e.g. subnational region), the subgroups with the highest and lowest estimates are used:

$$(12) \quad R = y_{max}/y_{min}.$$

Note that the selection of y_{max} and y_{min} depends on the type of health indicator and on the characteristics of the dimension of inequality, for which R is calculated.⁶ For place of residence, R is calculated as the ratio of urban to rural areas in the case of favourable health intervention indicators and as the ratio of rural to urban areas in the case of adverse health outcome indicators. For sex, R is calculated as the ratio of females to males in the case of favourable health intervention indicators and as the ratio of males to females in the case of adverse health outcome indicators. For dimensions

⁶ Selections were made based on convenience of data interpretation (that is, providing values above one for ratio calculations). In the case of sex, the selection does not represent an assumed advantage of one sex over the other.

with more than two subgroups that have a natural ordering, y_{max} refers to the most-advantaged subgroup and y_{min} to the most-disadvantaged subgroup in the case of favourable health intervention indicators, whereas y_{max} refers to the most-disadvantaged subgroup and y_{min} to the most-advantaged subgroup in the case of adverse health outcome indicators. For dimensions with more than two subgroups that have no natural ordering, the highest estimate is divided by the lowest estimate, regardless of the health indicator type.

R is calculated for all dimensions of inequality. In the case of dimensions with two subgroups or in the case of dimensions with more than two subgroups that have no natural ordering, R is missing if at least one subgroup estimate is missing. In the case of dimensions with more than two subgroups that have a natural ordering, R is missing if the estimates for the most-advantaged and/or most-disadvantaged subgroup are missing. Note that the bootstrap 95% confidence intervals for R are not calculated.

Interpretation

If there is no inequality, R takes the value one. It takes only positive values (larger or smaller than 1). The further the value of R from 1, the higher the level of inequality.

3.2.6 Relative concentration index

Definition

The relative concentration index (RCI) is a complex measure of inequality that shows the health gradient across multiple subgroups with natural ordering, on a relative scale. It indicates the extent to which a health indicator is concentrated among the disadvantaged or the advantaged.

Calculation

RCI is calculated by dividing the absolute concentration index (ACI) by the national average μ . This fraction may be more easily interpreted when divided by 100:

$$(13) \quad RCI = \frac{ACI}{\mu} * 100.$$

RCI is calculated for dimensions with more than two subgroups that have a natural ordering. It is missing if at least one subgroup estimate is missing.

Interpretation

RCI is bounded between -1 and +1 (or -100 and +100 if multiplied by 100) and takes the value zero if there is no inequality. Positive values indicate a concentration of the health indicator among the advantaged, while negative values indicate a concentration of the health indicator among the disadvantaged. The greater the absolute value of RCI, the higher the level of inequality.

3.2.7 Relative index of inequality

Definition

The relative index of inequality (RII) is a complex measure of inequality that represents the relative difference (proportional to the national level) in predicted values of health indicator between the most-advantaged and most-disadvantaged, while taking into consideration all the other subgroups – using an appropriate regression model.

Calculation

RII is calculated by dividing the slope index of inequality (SII) by the national average μ :

$$(14) \quad RII = \frac{SII}{\mu}.$$

RII is calculated for dimensions with more than two subgroups that have a natural ordering. It is missing if at least one subgroup estimate is missing. Note that the bootstrap 95% confidence intervals for RII are not calculated.

Interpretation

If there is no inequality, RII takes the value zero. Greater absolute values indicate higher levels of inequality. For favourable health intervention indicators, positive values indicate higher coverage in the advantaged subgroups and negative values indicate higher coverage in the disadvantaged subgroups. For adverse health outcome indicators, positive values indicate a higher concentration of the indicator among the disadvantaged and negative values indicate a higher concentration among the advantaged.

3.2.8 Theil index

Definition

The theil index (TI) is a complex measure of inequality that takes into account the population share of each subgroup.

Calculation

TI is calculated as the sum of products of the natural logarithm of the share of health of each subgroup ($\ln \frac{y_j}{\mu}$), the share of health of each subgroup ($\frac{y_j}{\mu}$) and the population share of each subgroup (p_j). TI may be more easily interpreted when multiplied by 1000:

$$(15) \quad TI = \sum_j p_j \frac{y_j}{\mu} \ln \frac{y_j}{\mu} * 1000,$$

where y_j indicates the estimate for subgroup j , p_j the population share of subgroup j and μ the national average.

TI is calculated for dimensions with more than two subgroups that have no natural ordering. It is missing if at least one subgroup estimate is missing. Note that the bootstrap 95% confidence intervals for TI are not calculated.

Interpretation

If there is no inequality, TI takes the value zero. Greater absolute values indicate higher levels of inequality. TI is more sensitive to health differences further from the national average (by the use of the logarithm).

Supplementary table 1 Study countries: ISO3 country codes, survey source(s) and year(s), WHO region and country income group

Country	ISO3 country code	Survey source(s) and year(s)	WHO Region	Country income group*
Afghanistan	AFG	MICS 2010–2011	Eastern Mediterranean	Low-income
Albania	ALB	DHS 2008–2009, MICS 2005	European	Middle-income
Argentina	ARG	MICS 2011–2012	Americas	Middle-income
Armenia	ARM	DHS 2010, DHS 2005, DHS 2000	European	Middle-income
Azerbaijan	AZE	DHS 2006	European	Middle-income
Bangladesh	BGD	MICS 2012–2013, DHS 2011, DHS 2007, MICS 2006, DHS 2004, DHS 1999–2000, DHS 1996–1997, DHS 1993–1994	South-East Asia	Middle-income
Barbados	BRB	MICS 2012	Americas	High-income
Belarus	BLR	MICS 2012, MICS 2005	European	Middle-income
Belize	BLZ	MICS 2011, MICS 2006	Americas	Middle-income
Benin	BEN	DHS 2011–2012, DHS 2006, DHS 2001, DHS 1996	African	Low-income
Bhutan	BTN	MICS 2010	South-East Asia	Middle-income
Bolivia (Plurinational State of)	BOL	DHS 2008, DHS 2003, DHS 1998, DHS 1994	Americas	Middle-income
Bosnia and Herzegovina	BIH	MICS 2011–2012, MICS 2006	European	Middle-income
Brazil	BRA	DHS 1996	Americas	Middle-income
Burkina Faso	BFA	DHS 2010, MICS 2006, DHS 2003, DHS 1998–1999	African	Low-income
Burundi	BDI	DHS 2010, MICS 2005	African	Low-income
Cambodia	KHM	DHS 2014, DHS 2010, DHS 2005, DHS 2000	Western Pacific	Middle-income
Cameroon	CMR	DHS 2011, MICS 2006, DHS 2004, DHS 1998	African	Middle-income
Central African Republic	CAF	MICS 2010, MICS 2006, DHS 1994–1995	African	Low-income
Chad	TCO	MICS 2010, DHS 2004, DHS 1996–1997	African	Low-income
Colombia	COL	DHS 2010, DHS 2005, DHS 2000, DHS 1995	Americas	Middle-income
Comoros	COM	DHS 2012, DHS 1996	African	Low-income
Congo	COG	DHS 2011–2012, DHS 2005	African	Middle-income
Costa Rica	CRI	MICS 2011	Americas	Middle-income
Cuba	CUB	MICS 2014, MICS 2010–2011, MICS 2006	Americas	Middle-income
Côte d'Ivoire	CIV	DHS 2011–2012, MICS 2006, DHS 1998, DHS 1994	African	Middle-income
Democratic Republic of the Congo	COD	DHS 2013–2014, MICS 2010, DHS 2007	African	Low-income
Djibouti	DJI	MICS 2006	Eastern Mediterranean	Middle-income
Dominican Republic	DOM	DHS 2013, DHS 2007, DHS 2002, DHS 1999, DHS 1996	Americas	Middle-income
Egypt	EGY	DHS 2014, DHS 2008, DHS 2005, DHS 2000, DHS 1995	Eastern Mediterranean	Middle-income
Ethiopia	ETH	DHS 2011, DHS 2005, DHS 2000	African	Low-income

Gabon	GAB	DHS 2012, DHS 2000	African	Middle-income
Gambia	GMB	DHS 2013, MICS 2005–2006	African	Low-income
Georgia	GEO	MICS 2005	European	Middle-income
Ghana	GHA	DHS 2014, MICS 2011, DHS 2008, MICS 2006, DHS 2003, DHS 1998, DHS 1993	African	Middle-income
Guatemala	GTM	DHS 1998–1999, DHS 1995	Americas	Middle-income
Guinea	GIN	DHS 2012, DHS 2005, DHS 1999	African	Low-income
Guinea-Bissau	GNB	MICS 2006	African	Low-income
Guyana	GUY	DHS 2009, MICS 2006–2007	Americas	Middle-income
Haiti	HTI	DHS 2012, DHS 2005–2006, DHS 2000, DHS 1994–1995	Americas	Low-income
Honduras	HND	DHS 2011–2012, DHS 2005–2006	Americas	Middle-income
India	IND	DHS 2005–2006, DHS 1998–1999	South-East Asia	Middle-income
Indonesia	IDN	DHS 2012, DHS 2007, DHS 2002–2003, DHS 1997, DHS 1994	South-East Asia	Middle-income
Iraq	IRQ	MICS 2011, MICS 2006	Eastern Mediterranean	Middle-income
Jamaica	JAM	MICS 2011, MICS 2005	Americas	Middle-income
Jordan	JOR	DHS 2012, DHS 2007, DHS 2002, DHS 1997	Eastern Mediterranean	Middle-income
Kazakhstan	KAZ	MICS 2010–2011, MICS 2006, DHS 1999, DHS 1995	European	Middle-income
Kenya	KEN	DHS 2008–2009, DHS 2003, DHS 1998, DHS 1993	African	Middle-income
Kyrgyzstan	KGZ	MICS 2014, DHS 2012, MICS 2005–2006, DHS 1997	European	Middle-income
Lao People's Democratic Republic	LAO	MICS 2011–2012, MICS 2006	Western Pacific	Middle-income
Lesotho	LSO	DHS 2009, DHS 2004	African	Middle-income
Liberia	LBR	DHS 2013, DHS 2007	African	Low-income
Madagascar	MDG	DHS 2008–2009, DHS 2003–2004, DHS 1997	African	Low-income
Malawi	MWI	MICS 2013–2014, DHS 2010, MICS 2006, DHS 2004, DHS 2000	African	Low-income
Maldives	MDV	DHS 2009	South-East Asia	Middle-income
Mali	MLI	DHS 2012–2013, DHS 2006, DHS 2001, DHS 1995–1996	African	Low-income
Mauritania	MRT	MICS 2011, MICS 2007	African	Middle-income
Mongolia	MNG	MICS 2010, MICS 2005	Western Pacific	Middle-income
Montenegro	MNE	MICS 2013, MICS 2005–2006	European	Middle-income
Morocco	MAR	DHS 2003–2004	Eastern Mediterranean	Middle-income
Mozambique	MOZ	DHS 2011, MICS 2008, DHS 2003, DHS 1997	African	Low-income
Namibia	NAM	DHS 2013, DHS 2006–2007, DHS 2000	African	Middle-income
Nepal	NPL	MICS 2014, DHS 2011, MICS 2010, DHS 2006, DHS 2001, DHS 1996	South-East Asia	Low-income
Nicaragua	NIC	DHS 2001, DHS 1997	Americas	Middle-income
Niger	NER	DHS 2012, DHS 2006, DHS 1998	African	Low-income
Nigeria	NGA	DHS 2013, MICS 2011, DHS 2008, MICS 2007, DHS 2003, DHS 1999	African	Middle-income

Pakistan	PAK	DHS 2012–2013, DHS 2006–2007	Eastern Mediterranean	Middle-income
Panama	PAN	MICS 2013	Americas	Middle-income
Peru	PER	DHS 2012, DHS 2011, DHS 2010, DHS 2009, DHS 2008, DHS 2007, DHS 2006, DHS 2005, DHS 2004, DHS 2000, DHS 1996	Americas	Middle-income
Philippines	PHL	DHS 2013, DHS 2008, DHS 2003, DHS 1998, DHS 1993	Western Pacific	Middle-income
Republic of Moldova	MDA	MICS 2012, DHS 2005	European	Middle-income
Rwanda	RWA	DHS 2010, DHS 2005, DHS 2000	African	Low-income
Saint Lucia	LCA	MICS 2012	Americas	Middle-income
Sao Tome and Principe	STP	DHS 2008–2009	African	Middle-income
Senegal	SEN	DHS 2014, DHS 2012–2013, DHS 2010–2011, DHS 2005, DHS 1997	African	Low-income
Serbia	SRB	MICS 2014, MICS 2010, MICS 2005–2006	European	Middle-income
Sierra Leone	SLE	DHS 2013, MICS 2010, DHS 2008, MICS 2005–2006	African	Low-income
Somalia	SOM	MICS 2006	Eastern Mediterranean	Low-income
South Africa	ZAF	DHS 1998	African	Middle-income
South Sudan	SSD	MICS 2010	African	Low-income
Sudan	SDN	MICS 2010	Eastern Mediterranean	Middle-income
Suriname	SUR	MICS 2010, MICS 2006	Americas	Middle-income
Swaziland	SWZ	MICS 2010, DHS 2006–2007	African	Middle-income
Syrian Arab Republic	SYR	MICS 2006	Eastern Mediterranean	Middle-income
Tajikistan	TJK	DHS 2012, MICS 2005	European	Middle-income
Thailand	THA	MICS 2005–2006	South-East Asia	Middle-income
The former Yugoslav Republic of Macedonia	MKD	MICS 2011, MICS 2005–2006	European	Middle-income
Timor-Leste	TLS	DHS 2009–2010	South-East Asia	Middle-income
Togo	TGO	DHS 2013–2014, MICS 2010, MICS 2006, DHS 1998	African	Low-income
Trinidad and Tobago	TTO	MICS 2006	Americas	High-income
Tunisia	TUN	MICS 2011–2012	Eastern Mediterranean	Middle-income
Turkey	TUR	DHS 2003, DHS 1998, DHS 1993	European	Middle-income
Turkmenistan	TKM	MICS 2006	European	Middle-income
Uganda	UGA	DHS 2011, DHS 2006, DHS 2000–2001, DHS 1995	African	Low-income
Ukraine	UKR	MICS 2012, DHS 2007, MICS 2005	European	Middle-income
United Republic of Tanzania	TZA	DHS 2010, DHS 2004–2005, DHS 1999, DHS 1996	African	Low-income
Uzbekistan	UZB	MICS 2006, DHS 1996	European	Middle-income
Vanuatu	VUT	MICS 2007–2008	Western Pacific	Middle-income
Viet Nam	VNM	MICS 2013–2014, MICS 2010–2011, MICS 2006, DHS 2002, DHS 1997	Western Pacific	Middle-income
Yemen	YEM	DHS 2013, MICS 2006	Eastern Mediterranean	Middle-income

Zambia	ZMB	DHS 2013–2014, DHS 2007, DHS 2001–2002, DHS 1996	African	Middle-income
Zimbabwe	ZWE	MICS 2014, DHS 2010–2011, MICS 2009, DHS 2005–2006, DHS 1999, DHS 1994	African	Low-income

DHS = Demographic and Health Survey; MICS = Multiple Indicator Cluster Survey.

* Country income group was determined using the World Bank classification as of September 2016 (available from: <http://data.worldbank.org/about/country-and-lending-groups>, accessed 17 January 2017).

Supplementary table 2 Summary measures of inequality: formulas and basic characteristics

Summary measure	Formula	Unit	Simple vs. complex measure	Ordered vs. non-ordered complex measure	Weighted vs. unweighted measure	Interpretation	Value of no inequality
Absolute measures							
Absolute concentration index	$ACI = \sum_j p_j (2X_j - 1)y_j$	Unit of indicator	Complex	Ordered	Weighted	Positive (negative) values indicate a concentration of the indicator among the advantaged (disadvantaged). The larger the absolute value of ACI, the higher the level of inequality.	Zero
Between-group variance	$BGV = \sum_j p_j (y_j - \mu)^2$	Squared unit of indicator	Complex	Non-ordered	Weighted	BGV takes only positive values with larger values indicating higher levels of inequality.	Zero
Difference	$D = y_{high} - y_{low}$	Unit of indicator	Simple	-	Unweighted	The larger the absolute value of D, the higher the level of inequality.	Zero
Mean difference from best performing subgroup	$MDB = \sum_j p_j y_j - y_{ref} $	Unit of indicator	Complex	Non-ordered	Weighted	MDB takes only positive values with larger values indicating higher levels of inequality.	Zero
Mean difference from mean	$MDM = \sum_j p_j y_j - \mu $	Unit of indicator	Complex	Non-ordered	Weighted	MDM takes only positive values with larger values indicating higher levels of inequality.	Zero
Population attributable risk	$PAR = y_{ref} - \mu$	Unit of indicator	Complex	Non-ordered	Weighted	PAR takes only positive values for favourable health intervention indicators and only negative values for adverse health outcome indicators. The larger the absolute value, the higher the level of inequality.	Zero
Slope index of inequality	$SII = v_1 - v_0$ for favourable health intervention indicators; $SII = v_0 - v_1$ for adverse health outcome indicators	Unit of indicator	Complex	Ordered	Weighted	For favourable (adverse) health indicators, positive values indicate a concentration among the advantaged (disadvantaged) and negative values indicate a concentration among the disadvantaged (advantaged). The larger the absolute value of SII, the higher the level of inequality.	Zero
Relative measures							
Index of disparity	$IDIS = \frac{1}{n} * \frac{\sum_j y_j - \mu }{\mu} * 100$	No unit	Complex	Non-ordered	Unweighted	IDIS takes only positive values with larger values indicating higher levels of inequality.	Zero
Kunst-Mackenbach index	$KMI = v_1/v_0$ for favourable health intervention indicators; $KMI = v_0/v_1$ for adverse health outcome indicators	No unit	Complex	Ordered	Weighted	KMI takes only positive values. The further the value of KMI from 1, the higher the level of inequality.	One
Mean log deviation	$MLD = \sum_j p_j (-\ln(\frac{y_j}{\mu})) * 1000$	No unit	Complex	Non-ordered	Weighted	The larger the absolute value of MLD, the higher the level of inequality.	Zero
Population attributable fraction	$PAF = \frac{PAR}{\mu} * 100$	No unit	Complex	Non-ordered	Weighted	PAF takes only positive values for favourable health indicators and only negative values for adverse health indicators. The larger the absolute value of PAF, the larger the degree of inequality.	Zero

Ratio	$R = y_{high}/y_{low}$	No unit	Simple	-	Unweighted	R takes only positive values. The further the value of R from 1, the higher the level of inequality.	One
Relative concentration index	$RCI = \frac{ACI}{\mu} * 100$	No unit	Complex	Ordered	Weighted	RCI is bounded between -1 and +1 (or -100 and +100 if multiplied by 100). Positive (negative) values indicate a concentration of the indicator among the advantaged (disadvantaged). The larger the absolute value of RCI, the larger the degree of inequality.	Zero
Relative index of inequality	$RII = \frac{SII}{\mu}$	No unit	Complex	Ordered	Weighted	For favourable (adverse) health indicators, positive values indicate a concentration among the advantaged (disadvantaged) and negative values indicate a concentration among the disadvantaged (advantaged). The larger the absolute value of RII, the higher the level of inequality.	Zero
Theil index	$TI = \sum_j p_j \frac{y_j}{\mu} \ln \frac{y_j}{\mu} * 1000$	No unit	Complex	Non-ordered	Weighted	The larger the absolute value of TI, the greater the level of inequality.	Zero

y_j = Estimate for subgroup j.

y_{high} = Estimate for subgroup high. Note that for place of residence, subgroup high refers to urban in the case of favourable health intervention indicators and to rural in the case of adverse health outcome indicators. For sex, subgroup high refers to females in the case of favourable health intervention indicators and to males in the case of adverse health outcome indicators. For dimensions with more than two subgroups that have a natural ordering, subgroup high refers to the most-advantaged subgroup in the case of favourable health intervention indicators and to the most-disadvantaged subgroup in the case of adverse health outcome indicators. For dimensions with more than two subgroups that have no natural ordering, subgroup high refers to the subgroups with the highest estimate. Note that reference subgroups for difference and ratio were selected based on convenience of data interpretation (that is, providing positive values for range difference calculations and values above one for range ratio calculations). In the case of sex, this does not represent an assumed advantaged of one sex over the other.

y_{low} = Estimate for subgroup low. Note that for place of residence, subgroup low refers to rural in the case of favourable health intervention indicators and to urban in the case of adverse health outcome indicators. For sex, subgroup high refers to females in the case of favourable health intervention indicators and to males in the case of adverse health outcome indicators. For dimensions with more than two subgroups that have a natural ordering, subgroup low refers to the most-disadvantaged subgroup in the case of favourable health intervention indicators and to the most-advantaged subgroup in the case of adverse health outcome indicators. For dimensions with more than two subgroups that have no natural ordering, subgroup low refers to the subgroup with the lowest estimate. Note that reference subgroups for difference and ratio were selected based on convenience of data interpretation (that is, providing positive values for range difference calculations and values above one for range ratio calculations). In the case of sex, this does not represent an assumed advantaged of one sex over the other.

y_{ref} = Estimate for reference group. Note that for place of residence, the reference group refers to urban. For dimensions with more than two subgroups that have a natural ordering, the reference group refers to the most-advantaged subgroup. For sex and for dimensions with more than two subgroups that have no natural ordering, the reference group refers to the subgroup with the highest estimate in the case of favourable health intervention indicators and to the subgroup with the lowest estimate in the case of adverse health outcome indicators.

p_j = Population share for subgroup j.

$X_j = \sum_j p_j - 0.5p_j$ = Relative rank of subgroup j.

μ = National average.

v_0 = Predicted value of the hypothetical person at the bottom of the social-group distribution (rank 0).

v_1 = Predicted value of the hypothetical person at the top of the social-group distribution (rank 1).

n = Number of subgroups.