Annex 1

MAXIMUM VARIATION IN PIEZOMETRIC LEVEL IN A TYPICAL UNDER-DRAINAGE SYSTEM

The accompanying figure shows the layout of a typical under-drainage system. The maximum variation in piezometric level of the filtered water occurs between points A and G of the bed. If, for simplicity, the turbulence in passing from point G to point H is omitted from the calculation, the variation in piezometric level is seen to be made up of three components:

(1) the resistance due to the subsurface flow of water from A to B through the layer of gravel surrounding the drain,
(2) the resistance and conversion of static head into velocity head as the water flows through the lateral drain from B to C, and
(3) the resistance and conversion of static head into velocity head as the water flows through the main drain from D to H.

Component (1)

The pressure losses due to component (1) may be calculated on the basis of Darcy's law, the resistance H_1 being obtained from the formula:

$$H_1 = \frac{1}{2\,kh} \times v_f \times \left(\frac{a}{2}\right)^2$$

where
v_f = filtration rate
a = distance between centres of lateral drains
k = coefficient of permeability of the gravel surrounding the lateral drains
h = thickness of gravel surround.

With gravel of 5 mm diameter, k is about 700 m/h (assuming 40% porosity). When h is 0.15 m, kh is approximately 100 m^2/h. With a high filtration rate of 0.5 m/h (to consider an extreme condition) and an interval between lateral drains of 1.5 m as shown in the figure, the resistance

$$H_1 = \frac{1}{2 \times 100} \times 0.5 \times \left(\frac{1.5}{2}\right)^2 = 0.0014 \text{ m} = 1.4 \text{ mm}.$$

Component (2)

The losses due to component (2) are calculated in the following way. In the lateral drain, length l, the flow increased linearly from B to C to a maximum value Q equal to the area drained multiplied by the rate of filtration, i.e.:

$$Q = a \times l \times v_f = 1.5 \times 10 \times 0.5 = 7.5 \text{ m}^3\text{/h} = 2.08 \times 10^{-3} \text{ m}^3\text{/s}.$$

A lateral drain 8 cm in diameter has a cross-sectional area of 5.03×10^{-3} m^2, so that the velocity of water in the drain at C is:

$$v = Q/A = (2.08 \times 10^{-3})/(5.03 \times 10^{-3}) = 0.414 \text{ m/s},$$

and the velocity head is:

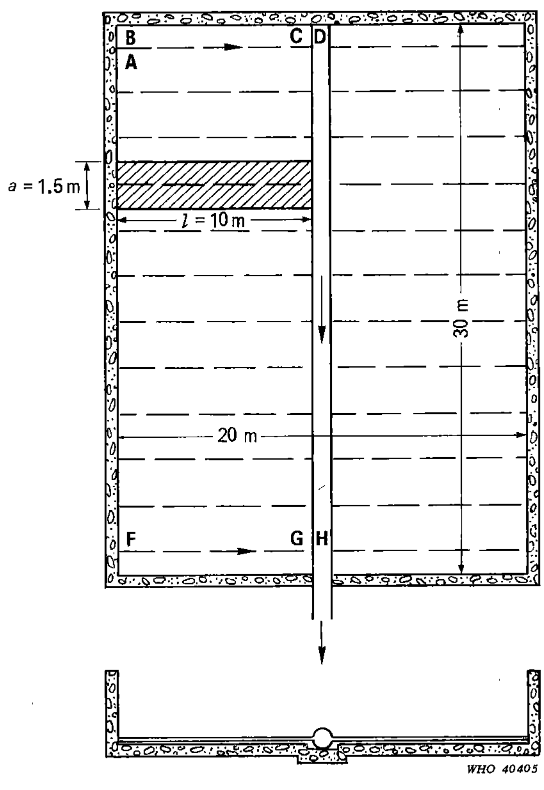
$$v^2/2g = 8.7 \times 10^{-3} \text{ m (since } g = 9.81 \text{ m/s}^2).$$

Given a linear increase in velocity from B to C, the resistance offered by a pipe of diameter d and length l is:

$$\frac{1}{3} \times \lambda \times \frac{l}{d} \times \frac{v^2}{2g}$$

where λ is the friction factor—a quantity obtained from Colebrook's formula: $1/\lambda = 2 \log (K/3.7d) + (2.51/R_e\sqrt{\lambda})$, in which K is the roughness of the pipe and R_e is the Reynolds number (derived from the kinematic viscosity at the particular temperature under consideration, the velocity of

CALCULATION OF THE LOSSES ACCOMPANYING THE FLOW OF WATER THROUGH THE UNDER-DRAINAGE SYSTEM



flow, and the diameter of the pipe). Since the Reynolds number varies with the velocity of flow, λ also varies along the length of the pipe. For a temperature of 10°C and a pipe roughness of 0.25 mm, λ ranges from 0.04 at one end of the pipe to 0.03 at the other, the average being about 0.033.

Thus, the resistance offered by the pipe is:

$$\frac{1}{3} \times 0.033 \times \frac{10}{0.08} \times \frac{v^2}{2g} = 1.37 \times \frac{v^2}{2g}$$

to which must be added a factor of $2v^2/2g$ representing the pressure loss due to the conversion of static head outside the drain to velocity head inside. This gives as the total change in piezometric level over the length BC:

$$H_2 = 3.37 \times v^2/2g = 3.37 \times 8.7 \times 10^{-3}\text{ m} = 30\text{ mm}.$$

Component (3)

The losses due to component (3), i.e., the fall in piezometric level due to the flow from D to H, may be calculated in a similar way. In the example under consideration (see figure), where the length l of the main drain is 30 m, the diameter d of the drain is 0.5 m, the width of the filter-bed is 20 m, and the filtration rate v_f is 0.5 m/h, the following quantities are readily calculated:

quantity carried, Q = area of filter-bed multiplied by filtration rate
$= 20 \times 30 \times 0.5 = 300\text{ m}^3/\text{h} = 8.33 \times 10^{-2}\text{ m}^3/\text{s}$
cross-sectional area of drain, $A = 19.6 \times 10^{-2}\text{ m}^2$
velocity of flow, $v = Q/A = (8.33 \times 10^{-2})/(19.6 \times 10^{-2}) = 0.425\text{ m/s}$.

The friction factor λ, obtained from Colebrook's formula, varies from 0.04 to 0.019, with an average value of 0.026. Thus, the resistance of the pipe over the length DH is:

$$\frac{1}{3} \times 0.026 \times \frac{30}{0.5} \times \frac{v^2}{2g} = 0.52 \times \frac{v^2}{2g}$$

to which must be added a further $2v^2/2g$ to convert the static head into velocity head. The total loss in piezometric level over the length DH then amounts to:

$$H_3 = 2.52 \times v^2/2g = 2.52 \times 9.2 = 23\text{ mm}.$$

The total pressure loss between A and G is now seen to be:

$$H_1 + H_2 + H_3 = 1.4 + 30 + 23 = 54.4\text{ mm} = 0.054\text{ m}.$$

In order that the variation over the area of the filter may be kept within an acceptable limit, say 10%, the total loss should not exceed 10% of the resistance of the filter-bed when at its lowest (i.e., when the sand is clean at the start of a filter run and when the bed is at its minimum thickness after repeated scrapings).

Under these conditions, with a depth of sand h of 0.75 m and a filtration rate v_f of 0.5 m/h, the filter-bed resistance H for a coefficient of permeability k is given by:

$$H = v_f h/k = 0.5 \times 0.75/k = 0.375/k.$$

If the maximum under-drainage losses (0.054 m) are to be kept within 10% of this value:

$$0.054 < 0.1 \times 0.375/k, \text{ or } k < 0.7 \text{ m/h}.$$

To obtain a coefficient of permeability of this value, a filter sand with an effective diameter of less than 0.15 mm is called for. If the sand is coarser than this, it will be necessary either to increase the diameters of the lateral and main drains or to lay the lateral drains more closely together. Since the variation in piezometric level is inversely proportional to between the fourth and fifth power of the pipe diameter, a small increase in the latter will make a large reduction in the head lost by the flowing water. It is therefore the remedy most commonly adopted. Similar measures will be necessary if it is desired to restrict the variation in filtration rate to a lower value than 10% (5% above and below average), but in view of the relatively small influence of filtration rate on effluent quality this precaution is seldom required.

Annex 2

PIEZOMETRIC LEVEL IN AN UNDER-DRAINAGE SYSTEM CONSTRUCTED WITH STANDARD BRICKS

There is a tendency today to use drainage systems other than pipes, except for the smallest filters. Fig. 10*a* (page 56) shows one of the simplest of such arrangements, using standard bricks to support the medium and to provide drainage space. If the dimensions of the bricks are $5 \times 11 \times 22$ cm, each channel drains a strip 23 cm wide and has the following hydraulic characteristics:

Cross-sectional area $A = 0.11 \times 0.18 = 1.98 \times 10^{-2}$ m^2
Wetted perimeter $P = 2 \times (0.11 + 0.18) = 0.52$ m
Hydraulic diameter $d = 4\, A/P = 4 \times 1.98 \times 10^{-2}/0.52 = 0.152$ m.

If an under-drainage system of this kind is used for the filter-bed shown in Fig. 9*b* (page 55), the length of the filter-bed being 60 m and the filtration rate being 0.5 m/h, the loss in piezometric level over the length of one channel may be calculated as follows:

$Q = 0.5 \times 60 \times 0.23 = 6.9$ m^3/h $= 1.92 \times 10^{-3}$ m^3/s
$v = Q/A = (1.92 \times 10^{-3})/(1.98 \times 10^{-2}) = 0.097$ m/s
$\lambda = 0.04$ (from Colebrook's formula assuming a roughness of 1 mm and a temperature of 10 °C)

$$\text{Thus resistance} = \frac{1}{3} \times 0.04 \times \frac{60}{0.152} \times \frac{v^2}{2\,g} = 5.3\, v^2/2\,g.$$

When the addition of $2v^2/2g$ is made for the conversion of static head into velocity head, the total head loss becomes $7.3 \times v^2/2g = 7.3 \times 0.48 = 3.5$ mm.

In such a large filter the main drain will be constructed in concrete and will probably be large enough to allow workmen to enter. A drain of this size involves a very low velocity of flow, and the drop in piezometric level will be less than 1 mm, thus giving a total head loss for the whole drainage system of $3.5 + 1.0 = 4.5$ mm.

With, as in Annex 1, a filtration rate of 0.5 m/h and a depth of sand-bed of 0.75 m, the variation in filtration rate throughout the filter-bed will be less than 10% when k is less than 8 m/h, or d_{10} is less than 0.45 mm. This requirement will nearly always be fulfilled, but it is nevertheless still good practice to check the hydraulic characteristics of the under-drainage system in every design. Other variations of the same principle are possible, and the engineer may exercise his ingenuity in devising the most suitable and economic system in the light of the materials locally available.